Reply by Author to A G Hammitt

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THE suggestion of Hammitt to replace the temperature by T^4 as the quantity that "slips" is reasonable from an empirical point of view when computing the temperature profiles in the intermediate regime for the particular problem considered The statement that the proposed boundary condition makes the accuracy of the heat-transfer result obtained in the original paper more "reasonable" for all mean free paths is open to question The important point is that it is *only* for small mean free paths that slip or temperature jump concepts have any theoretical justification In the case of small mean free paths, both the modification suggested by Hammitt and the original condition give the same answer This is easily seen by taking the fourth power of the original boundary condition and expanding it to give

$$T^4 = T_w^4 + Kl(dT^4/dy) + 0(l^2) +$$

Although the radiative heat-transfer problem becomes identical with the conductive heat-transfer problem between two parallel plates with the Maxwell-Smoluchowski temperature jump boundary condition[1] when one selects T^4 rather than T as the transfer parameter, this cannot be construed as an argument for the reasonableness of such an approach In the conductive heat-transfer problem, it is known that the heat-transfer result obtained is accurate even for large collision mean free paths (see, e g , Liu and Lees[2]) This good agreement is an empirical fact, although it is fortuitous One, therefore, cannot use the agreement as a justification in the case of the photon gas any more than it could be used with the molecular gas This was emphasized in the original paper, where it was pointed out that "there is little theoretical justification in extending Eq (9) to very small values of τ "

Hammitt points out a "limitation" of the radiation slip approach by noting that when it is applied directly to the Rayleigh problem it yields an initial heat transfer twice the blackbody value As Hammitt correctly notes, the radiative transfer problem is analogous to the low-density conduction problem In the usual low-density case, when temperature jump (or slip velocity) is applied directly to the Rayleigh problem, one obtains twice the correct heat transfer (or skin friction) in the infinite mean free path limit (see, e g , Ref 3) This factor of two is related to the fact that near the start of the motion in the Rayleigh problem very few photon collisions have occurred in the gas, so that one is close to the rarefied large mean free path case rather than the Rosseland limit It is not surprising, therefore, that a temperature or T^4 slip, based on the gradient of a mean quantity, yields an incorrect result This has, however, nothing to do with any limitation on the slip concept but only with its incorrect application It is no more possible to apply the radiation slip condition directly to the Rayleigh problem in a photon gas than it is to apply the usual slip or temperature jump conditions to this problem in a molecular gas

In regard to the separability of the radiative and conductive heat-transfer contributions, as noted by Hammitt, this is rigorously justifiable only on theoretical grounds in the opaque limit and in the large mean free path transparent limit The writer agrees with Hammitt that it cannot be defended on theoretical grounds for arbitrary mean free paths, no matter how small, in spite of the fact that it works rather well However, the separability of the radiative heat-transfer contribution is introduced in the same spirit as the slip approximation itself, i e , to provide a finite mean free path correction which goes over properly to the correct limits

It would be worthwhile to point out that the radiation slip concept has all the limitations and advantages that have been thoroughly discussed in the literature for the usual slip and temperature jump concepts in ordinary molecular kinetic theory

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